

# Exponential and Hyperbolic scalar-field in Tachyon cosmological models

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## Abstract

Tachyon scalar field in FRW universe considered, then scalar field and some important cosmological parameters for two special cases of scalar potential discussed. First we assume the exponential potential scalar field and then consider hyperbolic cosine type scalar-field potentials. In both cases we obtain behavior of the Hubble, deceleration and EoS parameters.

**Keywords:** Dark Energy, Cosmology.

## 1 Introduction

Recent cosmological observations [1-3] confirmed that our universe expanded while accelerated. Dark energy models come to explain nature of this accelerating expansion. The simplest model to describe the dark energy is the cosmological constant which has two famous problems as the fine-tuning and cosmic coincidence [4]. Hence, another models of the dark energy proposed such as Chaplygin gas model and its extensions [5-12].

There are also other interesting models such as quintessence [13-15], phantom [16, 17] and quintom [18-20]. These are based on the scalar fields which plays an important role in cosmology. One of the first major mechanisms where scalar fields are thought to be responsible is the inflationary scenario [21, 22]. Single scalar field is the underlying dynamics in many inflationary scenario. The mechanism of the initial inflationary and late-time acceleration aspect of our universe may be described by assuming the existence of some gravitationally coupled scalar fields, with the inflation field generating inflation and the quintessence field being responsible for the late accelerated expansion. Differences between models is an effective self-interaction potential. In the recent work [23], a new formalism for the analysis of scalar fields in flat isotropic and homogeneous cosmological models presented. Also,

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several new accelerating and decelerating exact cosmological solutions presented based on quintessence scalar field.

Recent discoveries [24, 25] have shown a number of novel and unexpected features, whose explanation will certainly require a deep change in our standard understanding of the universe. Therefore, this paper tries to study arbitrary scalar-field cosmology based on Tachyon cosmological models. The tachyon scalar field was proposed as a source of the dark energy and inflation. The tachyon dark energy has EoS parameter between -1 and 0 [26]. The tachyon is a an unstable field which has became important in string theory through its role in the Dirac-Born-Infeld action which is used to describe the D-brane action [27].

This paper is organized as follows. In next section we review Tachyon scalar field and in section 3 we write main equations which govern our model, and should solved for some specific potentials. In section 4 we obtain results corresponding to the exponential potential scalar field. In section 5 we obtain results corresponding to the Hyperbolic cosine type scalar-field. In section 6 we give conclusion.

## 2 Tachyon scalar field

Tachyon field described by the following energy density and pressure respectively [28, 29],

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}. \quad (1)$$

and,

$$p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2}, \quad (2)$$

Therefore, the equation of state of tachyon field obtained as follow,

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1. \quad (3)$$

Also Tachyon potential is given by,

$$V(\phi) = \sqrt{-p_\phi \rho_\phi}. \quad (4)$$

The general action of scalar-field models which minimally coupled to the gravitational field in the Einstein frame is given by,

$$S = \int d^4x \frac{\sqrt{-g}}{2} [R + g^{\mu\nu} \partial_\nu \phi \partial_\mu \phi - 2V(\phi)], \quad (5)$$

where  $R$  is the curvature scalar,  $\phi$  is the scalar field,  $V(\phi)$  is the self-interaction potential, and we used  $8\pi G = c = 1$  units.

### 3 FRW cosmology

The spatially flat Friedmann-Robertson-Walker (FRW) Universe is described by the following metric,

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2 d\Omega^2), \quad (6)$$

where  $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ . Also,  $a(t)$  represents time-dependent scale factor. Therefore, field equations obtained as follows,

$$3H^2 = \rho_\phi, \quad (7)$$

and

$$2\dot{H} + 3H^2 = -p_\phi, \quad (8)$$

where  $H = \dot{a}/a$  is Hubble expansion parameter, also  $\rho_\phi$  and  $p_\phi$  are given by the equations (1) and (2) respectively. Then, the evolution equation for the scalar field is given by,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (9)$$

where the over dot denotes the derivative with respect to the time-coordinate  $t$ , while the prime denotes the derivative with respect to the scalar field  $\phi$ , respectively.

Combining the equations (1), (2), (7) and (8) we can obtain the following equation describing evolution of Hubble expansion parameter,

$$\dot{H} = \frac{V^2(\phi)}{6H^2} - \frac{3H^2}{2}. \quad (10)$$

Also combining the equations (1), (7) and (9) we can obtain the following equation describing evolution of the scalar field,

$$\ddot{\phi} + \sqrt{\frac{3V(\phi)}{\sqrt{1 - \dot{\phi}^2}}} \dot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \quad (11)$$

The deceleration parameter  $q$  is an important observational quantity which is given by,

$$q = -\frac{\dot{H}}{H^2} - 1 = \frac{\dot{\phi}^2}{2} - 1. \quad (12)$$

In the case of constant  $\phi$  we yield  $q = -1$  which is de Sitter type accelerated universe. Now, we try to solve the above equations and extract cosmological parameters for some special cases of scalar field potential.

### 4 The exponential potential scalar field

The number of known exact solutions for cosmological models based on scalar fields is rather limited. One of such models is the flat Friedmann universe filled with a minimally coupled

scalar field with exponential potential. This solution describes a power-law expansion of the universe. The scalar potential is given by,

$$V = V_0 e^{\alpha\phi}, \quad (13)$$

where  $V_0$  and  $\alpha$  are arbitrary constants. In that case we have a condition as,

$$\frac{V'}{V} = \text{const.} \quad (14)$$

These types of exponential potential are important in four-dimensional effective Kaluza-Klein type theories from compactification of the higher-dimensional supergravity or superstring theories [30] and may arise due to non-perturbative effects such as gaugino condensation [31]. In that case, the equation (11) rewritten as the following form,

$$\ddot{\phi} + \sqrt{\frac{3V_0 e^{\alpha\phi}}{\sqrt{1 - \dot{\phi}^2}}} \dot{\phi} + \alpha V_0 e^{\alpha\phi} = 0. \quad (15)$$

Numerically, we find time evolution of the scalar field in the Fig. 1. We can see that it is decreasing function of time with negative value. Also, it is find that increasing  $\alpha$  increases net value of the scalar field.

It is general behavior of the scalar field. In order to obtain other important cosmological parameter we need an analytical expression of the scalar field.

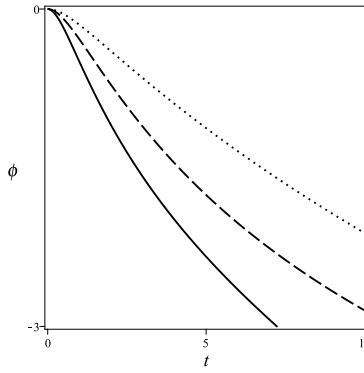


Figure 1: Scalar field in terms of  $t$  for  $V_0 = 1$ ,  $\alpha = 0.5$  (dotted line),  $\alpha = 1$  (dashed line),  $\alpha = 2$  (solid line).

If we assume  $\dot{\phi} \ll 1$  and  $\alpha \ll 1$ , then the explicit form of the scalar field will be available,

$$\phi = -\frac{\sqrt{3}}{3} \left[ C_1 \frac{e^{-\sqrt{3V_0}t}}{\sqrt{V_0}} + \sqrt{V_0}\alpha t \right] + C_2, \quad (16)$$

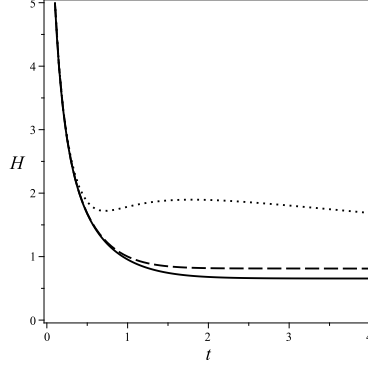


Figure 2: Hubble expansion parameter in terms of  $t$  for  $V_0 = 1$ ,  $C_1 = 5$ ,  $C_2 = 5$ ,  $\alpha = 0.5$  (dotted line),  $\alpha = 0.1$  (dashed line),  $\alpha = 0.01$  (solid line).

where  $C_1$  and  $C_2$  are integration constants. Using this approximation we can obtain other cosmological parameters such as the deceleration parameter,

$$q = -1 + \frac{1}{2} \left( C_1 e^{-\sqrt{3V_0}t} - \sqrt{\frac{V_0}{3}}\alpha \right)^2. \quad (17)$$

It is easy to find that the deceleration parameter is decreasing function of time yields to -1 at the late time. Corresponding to the value of the constant  $C_1$ , it is possible to see accelerating to decelerating phase transition.

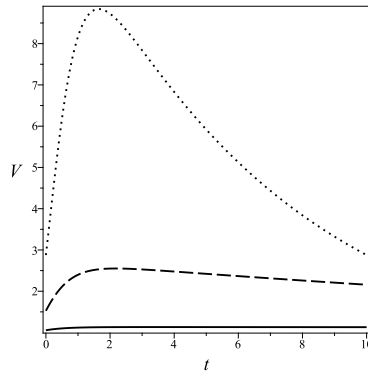


Figure 3: Scalar potential in terms of  $t$  for  $V_0 = 1$ ,  $C_1 = 5$ ,  $C_2 = 5$ ,  $\alpha = 0.5$  (dotted line),  $\alpha = 0.2$  (dashed line),  $\alpha = 0.025$  (solid line).

We can also investigate numerically, behavior of the Hubble expansion parameter. In the Fig. 2 we draw Hubble expansion parameter versus cosmic time. We can see that it is decreasing function of time which yields to a constant value at the late time which is

expected. It is illustrated that the value of the Hubble parameter decreased by  $\alpha$ . Also, we can obtain equation of state parameter as the follow,

$$\omega = \left( C_1 e^{-\sqrt{3V_0}t} - \sqrt{\frac{V_0}{3}}\alpha \right)^2 - 1, \quad (18)$$

which guarantee that  $\omega \geq -1$ . It is clear that the EoS parameter yields to -1 at the late time.

Finally, we can see behavior of the scalar potential in the Fig. 3. For the very small values of parameter  $\alpha$  the scalar potential behaves as a constant. Generally we have a maximum for the potential and it yields to zero at the late time.

## 5 Hyperbolic cosine type scalar-field potentials

It is also possible to consider the hyperbolic cosine type scalar-field potentials given by,

$$V = V_0 \cosh^\beta[\gamma(\phi - \phi_0)], \quad (19)$$

where  $V_0$ ,  $\beta$ ,  $\gamma$  and  $\phi_0$  are arbitrary constants. In that case the condition (14) is no longer valid. Therefore, the equation (11) rewritten as the following form,

$$\ddot{\phi} + \sqrt{\frac{3V_0 \cosh^\beta[\gamma(\phi - \phi_0)]}{\sqrt{1 - \dot{\phi}^2}}} \dot{\phi} + \frac{dV}{d\phi} = 0. \quad (20)$$

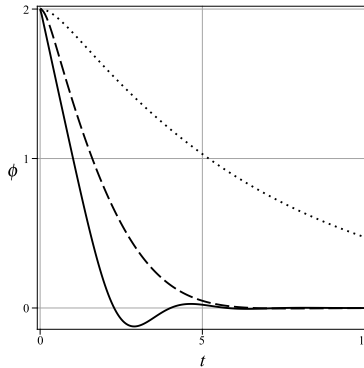


Figure 4: Scalar field in terms of  $t$  for  $\phi_0 = 0$ ,  $V_0 = 1$ ,  $\gamma = 0.5$  (dotted line),  $\gamma = 1$  (dashed line),  $\gamma = 2$  (solid line).

General solution of the equation (20) presented in Fig. 4 numerically. Similar to the previous case we can see that the scalar field is decreasing function of time but opposite the previous case it can has positive value. It is illustrated that the scalar field yields to a

constant at the late time. For larger values of  $\gamma$  than 1 the scalar field takes negative value some time.

If we assume  $\dot{\phi} \ll 1$  and  $\gamma \ll 1$ , then the explicit form of the scalar field will be available,

$$\phi = C_1 e^{-\frac{\sqrt{3}}{6}(3\sqrt{V_0}-\sqrt{9V_0-12V_0\gamma^2})t} + C_2 e^{-\frac{\sqrt{3}}{6}(3\sqrt{V_0}+\sqrt{9V_0-12V_0\gamma^2})t}, \quad (21)$$

where  $C_1$  and  $C_2$  are integration constants. Using this approximation we can obtain other cosmological parameters such as the deceleration parameter. It is easy to find that the deceleration parameter is decreasing function of time yields to -1 at the late time similar to the previous model. It is also possible to see accelerating to decelerating phase transition.

We can perform numerical analysis on the Hubble expansion parameter. In the Fig. 5 we draw Hubble expansion parameter versus cosmic time. We can see that it is decreasing function of time which yields to a constant value at the late time which is expected. It is illustrated that the value of the Hubble parameter decreased by  $\gamma$ .

Also we can obtain equation of state parameter and find that  $\omega \geq -1$ . It is clear that the EoS parameter yields to -1 at the late time.

Finally, we can see behavior of the scalar potential in the Fig. 6. For the very small values of parameter  $\gamma$  the scalar potential behaves as a constant. Generally, the potential is decreasing function of time which is different with the previous model.

We found reasonable behavior of our models, however we have to add matter to the models and compare our results with appropriate observational data such as  $H(z)$  data [32].

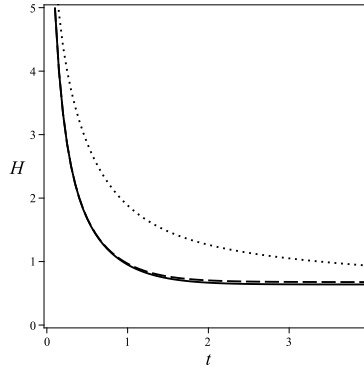


Figure 5: Hubble expansion parameter in terms of  $t$  for  $V_0 = 1$ ,  $C_1 = 5$ ,  $C_2 = 5$ ,  $\gamma = 0.5$  (dotted line),  $\gamma = 0.1$  (dashed line),  $\gamma = 0.01$  (solid line).

## 6 Matter contribution

In order to have comparison with observational data we should consider matter contribution in our model. In that case we have the following conservation equation,

$$\dot{\rho} + 3H(p + \rho) = 0, \quad (22)$$

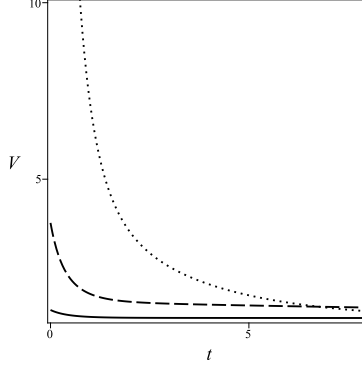


Figure 6: Scalar potential in terms of  $t$  for  $V_0 = 1$ ,  $C_1 = 5$ ,  $C_2 = 5$ ,  $\gamma = 0.5$  (dotted line),  $\gamma = 0.2$  (dashed line),  $\gamma = 0.075$  (solid line).

where  $\rho = \rho_\phi + \rho_m$  and  $p = p_\phi + p_m$ , with matter density  $\rho_m$  and pressure  $p_m$ . Now, the equations (7) and (8) extended to the following relations,

$$3H^2 = \rho, \quad (23)$$

and,

$$2\dot{H} = -p - \rho. \quad (24)$$

We assume non-interacting case, therefore conservation equation (22) separates as follow,

$$\dot{\rho}_\phi + 3H(p_\phi + \rho_\phi) = 0, \quad (25)$$

and

$$\dot{\rho}_m + 3H(p_m + \rho_m) = 0, \quad (26)$$

with  $\omega_m = p_m/\rho_m$  as EoS of matter. An important parameter to compare with observational data is  $H(z)$ . We will calculate  $H(z)$  for two different cases of exponential and Hyperbolic scalar potential.

## 6.1 Exponential potential

In this case we use relation (13) and investigate behavior of Hubble expansion parameter versus redshift. We can see good agreement with observational data for  $0.01 \leq \alpha \leq 0.5$ . However,  $0.1 < \alpha < 0.5$  yields to the best agreement with high redshift at the early universe.

## 6.2 Hyperbolic potential

In this case we use relation (19) and investigate behavior of Hubble expansion parameter versus redshift. We can see good agreement with observational data for  $0.1 \leq \gamma \leq 0.5$ .

Comparison of both models with each other suggest that the second model (Hyperbolic potential) is more appropriate.



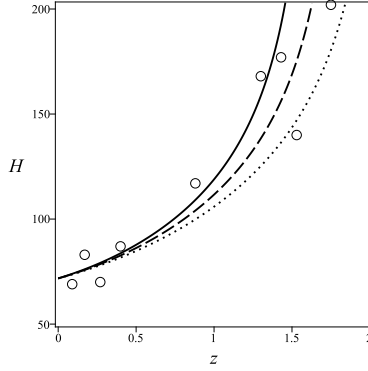


Figure 7: Hubble expansion parameter in terms of  $z$  for  $V_0 = 1$ ,  $C_1 = 5$ ,  $C_2 = 5$ ,  $\alpha = 0.5$  (dotted line),  $\alpha = 0.1$  (dashed line),  $\alpha = 0.01$  (solid line). Big dots denote observational data.

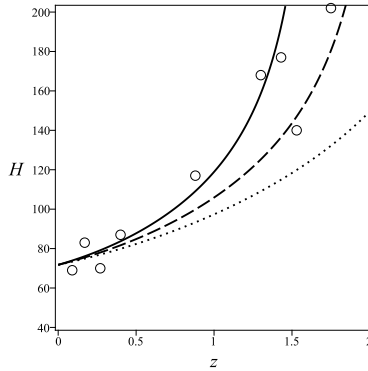


Figure 8: Hubble expansion parameter in terms of  $z$  for  $V_0 = 1$ ,  $C_1 = 5$ ,  $C_2 = 5$ ,  $\gamma = 0.5$  (dotted line),  $\gamma = 0.1$  (dashed line),  $\gamma = 0.01$  (solid line). Big dots denote observational data.

## 7 Conclusion

In this work, we considered Tachyon scalar field in FRW universe and proposed two different model based on scalar field potential. In the first model, we assumed the exponential potential scalar field and solved the equation describing evolution of the scalar field numerically. We found that the scalar field in this model has totally negative value and decreased by time. It means that at the late time the scalar field takes negatively infinite value. In order to obtain other cosmological parameters we assumed infinitesimal tile evolution of scalar field and found explicit expression for the scalar field, deceleration and EoS parameters. Also we discussed numerically about tachyon potential and Hubble expansion parameter. We found that the deceleration parameter as well as EoS parameter is decreasing function of time and yields to -1 at the late time. We have shown that the Hubble expansion parameter

is decreasing function of time and yields to a constant value at the late time as expected. Finally we found corresponding to the very small values of parameter  $\alpha$  the scalar potential behaves as a constant. In general, there is a maximum for the potential and it yields to zero at the late time. Apart of the unlike behavior of the scalar field, the model yields to good results. It yields us to investigate another model to obtain also good behavior for the scalar field. In the second model the hyperbolic cosine type scalar-field potentials considered. Our numerical analysis shown that the scalar field is decreasing function of time with the positive value. It is illustrated that the scalar field yields to a constant at the late time. It is more suitable than the first model and yields to good behavior cosmological parameters like Hubble, deceleration and EoS parameters. Therefore, we can suggest the second model as a good model to describe universe. In order to confirm our claim we added matter to the model obtained Hubble expansion parameter in terms of redshift and compare our results with observational data to find that the second model is in agreement with observational data.

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